OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4751:** Introduction to Advanced Mathematics

Mark Scheme for January 2012

Oxford Cambridge and RSA Examinations

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations

Annotation in scoris	**Meaning**
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awar marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark schem in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not b answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the workin looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect r must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, awa the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involve your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candida intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem i substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be giver associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or ex th of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case w passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the schem otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a part dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously inco Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accura given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in th rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-c

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates oth are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) sh penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The s particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rati contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then exa the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mar[illegible] appears (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question ren[illegible] according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidan	
1			grad = –1/5 oe	M1		allo
			$y - 6$ = their m $(x - 1)$ or 6 = their m [× 1] + c	M1		if first M1 i M1 for $y - 6$ = number excep
			$y = -0.2x + 6.2$ oe isw	A1	terms collected, with y as subject or for $a = -0.2$, $b = 6.2$ oe	allow A1 for c oe already see condone $y = -$
				[3]		
2	(i)		$\frac{1}{3}$ as final answer	2 **[2]**	allow $\pm\frac{1}{3}$ M1 for $\frac{1}{9^{\frac{1}{2}}}$ or for $9^{\frac{1}{2}} = \sqrt{9}$ or 3 soi	eg M1 for 3^{-1}
2	(ii)		$32x^{10}y^{-3}$ or $\frac{32x^{10}}{y^3}$ oe as final answer	3 **[3]**	B1 for each element if B0, allow M1 for $(4x^4)^3 = 64x^{12}$	allow 2^5 instea
3			$6n^2 + 12n + 8$ or $2(3n^2 + 6n + 4)$ oe as final answer	3 **[3]**	B2 for 2 terms correct in final answer or for $(n + 2)^3 = n^3 + 6n^2 + 12n + 8$ or B1 for 1, 3, 3, 1 soi or SC2 for final answer of $3n^2 + 6n + 4$	B1 for $n^3 + 4n^2 + 4n$ condoning one

Question			Answer	Marks	Guidan	
4	(i)		$23+\sqrt{2}$ as final answer	3 **[3]**	B2 for 23 and B1 for $\sqrt{2}$ or $1\sqrt{2}$ or M2 for 3 or more terms correct of $35-14\sqrt{2}+15\sqrt{2}-12$ or M1 for 2 terms correct	m he mixt eg M2 for
4	(ii)		$5\sqrt{6}$ isw	2 **[2]**	condone $\frac{30}{\sqrt{6}}$ for 2 marks M1 for $\left[\sqrt{54}=\right]3\sqrt{6}$ or $\left[\frac{12}{\sqrt{6}}=\right]2\sqrt{6}$	eg 2 isw for 5

Question			Answer	Marks	Guidan	
5			$6(2x + 1) < 5(3x + 4)$	M1	for multiplying up correctly or for correct use of a common denominator	fi 's equa condone o then expande
			$12x + 6 < 15x + 20$ or ft	M1	for expanding brackets correctly; for combined first two steps with one error, such as $12x + 6 < 15x + 4$, allow M1M0	eg $\frac{12x+6}{30} < \frac{1}{}$
			$-14 < 3x$ or $-3x < 14$ or ft	M1	for collecting terms correctly	ft from two x t
			$x > -\frac{14}{3}$ oe or ft isw	M1	for final division of their inequality with ax on one side, $a \neq 1$ or 0, and non-zero number on the other allow SC3 for $-14/3$ found without correct inequality symbol(s)	allow working making correc eg allow last M $\frac{-14}{3} < x$ isw reminder : (−1 notation
			or $\frac{1}{5} - \frac{4}{6} < \frac{3x}{6} - \frac{2x}{5}$ oe	or M1		
			$\frac{-7}{15} < \frac{3x}{30}$ oe or ft	M2	M1 for one side correct ft	
			$x > -\frac{14}{3}$ oe or ft isw	M1	as in previous method	
				[4]		

Question			Answer	Marks	Guidan	
6			$4h + ha^2 = 9a - 5$ $h(4 + a^2) = 9a - 5$ $[h =]\dfrac{9a-5}{4+a^2}$ oe as final answer	M1 M1 M1 **[3]**	correctly collecting h terms on one side, remaining terms on other for factorising, ft eg sign error for division by their factor; ft only for equiv difficulty	M0 if seen and 'cancelling'
7	(i)		'tick' at (2,4)(3,1)(5,6)	2 **[2]**	mark intent M1 for two points correct or for 'tick' at (2,–2) (3,–5) and (5,0)	overlay to be p condone tick u allow M1 for p correct:
7	(ii)		'tick' at (0,1)(1,–2)(3,3)	2 **[2]**	mark intent M1 for two points correct or for 'tick' at (4,1) (5,–2) and (7,3)	overlay to be p condone tick u allow M1 for p correct:
8			$5(x + 1.5)^2 + 0.75$ oe www 0.75 oe or ft their c	4 1 **[5]**	B1 for $a = 5$ and B1 for $b = 3/2$ oe and B2 for $c = 3/4$ oe or M1 for $12 - 5 \times (\text{their } 3/2)^2$ oe soi or for $2.4 - (\text{their } 3/2)^2$ oe [eg 0.15] soi 0 for (–1.5, 0.75)	condone omis eg $5[(x + 7.5)^2$ B1B0M1ft condone found differentiation

Question			Answer	Marks	Guidan	
9	(i)		'if n even then n^3 even, so $n^3 + 1$ odd' oe	B1	must mention n^3 is even or even^3 is even or even × even = even	0 [illegible] n 0 if j[illegible]
			$\Leftarrow$ with if $n^3 + 1$ odd then n^3 even but if n^3 is even, n is not necessarily an integer or $\Leftrightarrow$ with '$n^3 + 1$ odd then n^3 even so n even', [assuming n is an integer]	B1	or '$\Leftrightarrow$ with if n is odd, n^3 is odd, so $n^3 + 1$ is even' if 0 in question, allow SC1 for $\Leftrightarrow$ or $\Leftarrow$ and attempt at using general odd/even in explanation	condone $\leftarrow$ [illegible] parts must go furthe[illegible] in the qn; please annotat[illegible]
				[2]		
9	(ii)		showing $\Leftarrow$ is true	B1	eg when $x > 3$, +ve × +ve > 0	0 for just exan[illegible] stating it is tru[illegible]
			$\Leftarrow$ chosen and showing that $\Rightarrow$ [and therefore $\Leftrightarrow$] is/ are not true	B1	stating that true when $x < 2$ or giving a counterexample such as 1, 0 or a negative number [to show quadratic inequality also true for this number] allow B2 for $\Leftarrow$ and $x > 3$ and $x < 2$ shown/stated as soln or sketch showing two solns of $x^2 - 5x + 6 > 0$	0 for saying ar[illegible] or B1 for this [illegible] symbol
				[2]		

Question			Answer	Marks	Guidan	
10	(i)		grad AB = $\frac{7-1}{4-2}$ oe or 3	M1		
			$y - 7$ = their m $(x - 4)$ or $y - 1$ = their m $(x - 2)$	M1	or use of y = their gradient $x + c$ with coords of A or B or M2 for $\frac{y-1}{7-1} = \frac{x-2}{4-2}$ o.e.	allow ste or eg M1 for c then M1 for m and c
			$y = 3x - 5$ oe	A1	accept equivalents if simplified eg $3x - y = 5$ allow B3 for correct eqn www	allow A1 for already seen B2 for eg $y -$
				[3]		
10	(ii)		showing grad BC = $\frac{2-1}{-1-2} = -\frac{1}{3}$ oe	B1	may be calculation or showing on diagram	
			and $-1/3 \times 3 = -1$ or grad BC is neg reciprocal of grad AB, [so 90°]	B1	may be earned for statement / use of $m_1m_2 = -1$ oe, even if first B1 not earned for B1+B1, must be fully correct, with 3 as gradient in (i)	eg allow 2nd B = −1/3 with no earned
			<u>or</u> for finding AC or AC^2 independently of AB and BC	<u>or</u> B1	working needed such as $AC^2 = 5^2 + 5^2 = 50$	condone any c squares and sq B1 and for two $= \sqrt{50}$
			for correctly showing $AC^2 = BC^2 + AB^2$ oe	B1	working needed using correct notation such as $BC^2 = 3^2 + 1^2 = 10$; $AB^2 = 6^2 + 2^2 = 40$, $40 + 10 = 50$ [hence $AC^2 = BC^2 + AB^2$]	accept eg 3 an and $BC^2 = 10$ 0 for eg $\sqrt{40}$ +

Question			Answer	Marks	Guidan	
			or finding equation of line through C perpendicular to AB ($y = -\frac{1}{3}x + \frac{5}{3}$ oe)	or B1		eg B
			showing B is on this line either by substitution or finding intersection of this line with AB	B1		or B1 for find line through B oe and B1 for lines and show
			BC = $\sqrt{3^2 + 1^2}$ or $\sqrt{10}$	M1		
			AB=$\sqrt{6^2 + 2^2}$ or $\sqrt{40}$ or $2\sqrt{10}$	M1	both these Ms may be earned earlier if Pythag used to show angle ABC = 90°, but are for BC and AB, not BC^2 and AB^2	for both M1s a equivs
			Area = 10 [square units]	A1	must be simplified to 10	
			or area under AC – area under AB – area under BC	or M1		mark equivale methods, eg tr method, omitt ½ × 7 × 5 – (½ = 17.5 – (1.5
			at least two of 22.5, 8 and 4.5 oe	M1		
			Area = 10 [square units]	A1	must be simplified to 10	
				[5]		

Question			Answer	Marks	Guidan	
10	(iii)		(1.5, 4.5) oe angle in semicircle oe is a right-angle [so B is on circle] and must mention AC as diameter or D as centre [hence A, B, C all same distance from D]	2 E1 **[3]**	B1 each coordinate or '[since b = 90°,] ABC are three vertices of a rectangle. D is the midpoint of one diagonal <u>and</u> so D is the centre of the rectangle <u>or</u> the diagonals of a rectangle are equal and bisect each other, [hence DA=DB=DC] or condone showing that line from D to mid point of AB is perp to AB, so DBA is isos [hence DB = DA = DC] [or equiv using DBC]	E0 for j hypotenus so DAB is iso it is isw eg wrong NB some wro is isos

Question			Answer	Marks	Guidan	
11	(i)		f(−3) used	M1		
			−54 − 27 + 69 + 12 [= 0] isw	A1	or M1 for correct division by $(x + 3)$ or for the quadratic factor found by inspection and A1 for concluding that $x = -3$ [is a root] (may be earned later)	A0 for conclu factor
			attempt at division by $(x + 3)$ as far as $2x^3 + 6x^2$ in working	M1	or inspection with at least two terms of three–term quadratic factor correct; or at least one further root found using remainder theorem	
			correctly obtaining $2x^2 - 9x + 4$	A1	or stating further factor, found from using remainder theorem again	
			factorising the correct quadratic factor	M1	for factors giving two terms of quadratic correct or for factors ft one error in quadratic formula or completing square; M0 for formula etc without factors found	allow for $(x -$ factors eg afte theorem again
			$(2x - 1)(x - 4)[(x + 3)]$ isw	A1	allow $2(x - ½)$ instead of $(2x - 1)$, oe condone inclusion of '= 0'	isw $(x - ½)$ as found, even if
				[6]		

Question			Answer	Marks	Guidan	
11	(ii)		sketch of cubic right way up, with two turning points	B1	0 if stops at x-axis ignore graph of $y = 4x + 12$	m ru (exc 0.5); co ends but n points; must allow max on quadrants condone some
			values of intns on x axis shown, correct (−3, 0.5 and 4) or ft from their factors or roots in (i)	B1	on graph or nearby in this part mark intent for intersections with both axes	allow if no gra condone 3 on condone eg 0. between their
			12 marked on y-axis	B1 **[3]**	or $x = 0$, $y = 12$ seen in this part if consistent with graph drawn	allow if no gra with intn on − near their indi
11	(iii)		$2x^3 - 3x^2 - 23x + 12 = 4x + 12$ oe	M1	or ft their factorised f(x)	
			$2x^3 - 3x^2 - 27x$ [= 0]	A1	after equating, allow A1 for cancelling $(x + 3)$ factor on both sides and obtaining $2x^2 - 9x$ [= 0]	condone slip c
			[x]$(2x - 9)(x + 3)$ [= 0]	M1	for linear factors of correct cubic, giving two terms correct or for quadratic formula or completing square used on correct quadratic $2x^2 - 3x - 27 = 0$, condoning one error in formula etc;	or after cancel M1 for $x(2x -$ or 9/2 oe M0 for eg qua cubic, unless r given
			[x =] 0, −3 and 9/2 oe	A1 **[4]**	need not be all stated together	eg $x = 0$ may b

Question			Answer	Marks	Guidan	
12	(i)		$\sqrt{20}$ isw or $2\sqrt{5}$ (2, 0)	B1 B1 **[2]**	0 for $\pm\sqrt{20}$	
12	(ii)		subst of $x = 0$ into circle eqn soi	M1	or Pythag used on sketch of circle: $2^2 + y^2 = 20$ oe	M0 for just y for $y = 4$ ignore intns w
			$y = \pm 4$ oe	A1	or B2 for just $y = \pm 4$ seen oe; accept both 4 and –4 shown on y axis on sketch if both values not stated	
			sketch of circle with centre (2, 0) or ft their centre from (i)	B1 **[3]**	if the centre is not marked, it should look roughly correct by eye – coords need not be given on sketch; condone intersections with axes not marked	circle should i neg x- and y-a attempt at circ ignore any tan
12	(iii)		$(x - 2)^2 + (2x + k)^2 = 20$	M1	for attempt to subst $2x + k$ for y	allow for atten into given eqn
			$x^2 - 4x + 4 + 4x^2 + 4kx + k^2 = 20$	M1 dep	for correct expansion of at least one set of brackets, dependent on first M1	similarly for t
			$5x^2 + (4k - 4)x + k^2 - 16 = 0$	A1 **[3]**	correct completion to given answer; dependent on both Ms	condone omis step if both se correctly, but backwards, at needed; if cands have to correct it, c complete to a

Question			Answer	Marks	Guidan	
12	(iv)		$b^2 - 4ac = 0$ seen or used	M1	need not be substituted into; may be stated after formula used or argument towards expressing eqn as a perfect square	eg $(4k -$
			$4k^2 + 32k - 336$ [= 0] or $k^2 + 8k - 84$ [= 0]	M1	expansion and collection of terms, condoning one error ft their $b^2 - 4ac$	dep on an atte least two of a, earned with <
			use of factorising or quadratic formula or completing square	M1	condone one error ft	dep on attemp quadratic equa with any eqn/i
			$k = 6$ or -14 or Grad of tgt is 2, and normal passes through centre, hence finding equation of normal as $y = -\frac{1}{2}x + 1$ oe	A1 or M1		
			finding x values where diameter $y = -x/2 + 1$ intersects circle as $x = 6$ or -2 (condone one error in method)	M1	oe for y values; condone one error in method	or finding intn $\left(\frac{2-2k}{5}, \frac{k+4}{5}\right.$
			finding corresponding y values on circle and subst into $y = 2x + k$ or subst their x values into $5x^2 + (4k - 4)x + k^2 - 16 = 0$	M1	intns are (6, –2) and (–2, 2), M0 for just (6, 2) and (–2, –2) used but condone used as well as correct intns this last method gives extra values for k, for the non-tangent lines $y =$ through (6, 2) and (–2, –2), but allow for the M mark	or subst their i into eqn of cir $\left(\frac{2-2k}{5} - 2\right)^2$
			$k = 6$ or -14	A1 **[4]**	and no other values	

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